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DISCUSSIONS.

THE DOCTRINE OF CONSERVATION OF ENERGY IN ITS RELATION TO THE ELIMINATION OF FORCE AS A FACTOR IN THE COSMOS.

The modern theory of conservation of energy may be said to date from 1842, when Dr. Julius Robert Mayer, of Heilbronn, formally stated that there is a connexion between heat and work, and introduced into the language of physics the expression "mechanical equivalent of heat." The investigations of Joule, Thomson, Rankine, Helmholtz, Clausius, Holtzmann, Clapeyron, and Regnault, may be said to have established the principle, formulated by Prof. James Clerk Maxwell as follows:

"*The total energy of any body or system of bodies is a quantity which can neither be increased nor diminished by any mutual action of those bodies, though it may be transformed into any one of the forms of which energy is susceptible.*"

The doctrine is now generally regarded, among mathematicians and physicists, as established beyond the possibility of being overturned. Granting its truth as stated, it assumes several fundamental notions as axioms or postulates upon which to base the deduction. They are as follows:

1. Space, or extension in three dimensions.
2. Time, or duration.
3. Matter, with the property of being increate and indestructible.
4. Motion, or translation in space.
5. Force, that which opposes or tends to produce motion in matter,—which may produce motion in matter or merely stress, a tendency to motion.
6. Mass,[1] or inertia, that property of matter by which it resists the action of a

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[1] This sense of the term "mass" is that in which it is used by Professor Maxwell, who has done as much, if not more, than any other writer, to give us a rational working theory of molecular statics and dynamics,—one which avoids the necessity of assuming action at a distance. The common understanding of "mass" is the "amount of matter." But the amount of matter is always in direct ratio of inertia; so the common notion may be considered as included under the more comprehensive notion of Mr. Maxwell.

force,—such resistance as requires time for the production or destruction of motion in matter.

Space and time are generally regarded as fundamental notions in the nature of intuitions and requiring no explanation. The indestructibility of matter and the correlative proposition that matter cannot be created are now also generally conceded, though not axiomatic or susceptible of proof, except by limited experimentation. Investigation has been sufficiently extensive, however, to establish the indestructibility of matter as far as human powers and observation of natural forces go. Motion is a postulate which no one denies, except those idealists who deny the credibility of the senses. Inertia, or mass, is generally assumed as a property of matter requiring no proof or explanation, except that through the media of the senses by experimentation. However, Prof. James Clerk Maxwell says on this point;

> "But one of the first, if not the very first, desideratum in a complete theory of matter is to explain—first, mass; and second, gravitation. To explain mass may seem an absurd achievement. We generally suppose that it is the essence of matter to be the receptacle of momentum and energy, and even Thomson, in his definition of his primitive fluid, attributes to it the possession of mass. But according to Thomson, though the primitive fluid is the only true matter, yet that which we call matter is not the primitive fluid itself, but a mode of motion of that primitive fluid. It is the mode of motion which constitutes the vortex rings, and which furnishes us with examples of that permanence and continuity of existence which we are accustomed to attribute to matter itself. The primitive fluid, the only true matter, eludes our perceptions when it is not endued with the mode of motion which converts certain portions of it into vortex rings, and thus renders it molecular.
>
> "In Thomson's theory, therefore, the mass of bodies requires explanation. We have to explain the inertia of what is only a mode of motion, and inertia is a property of matter, not modes of motion."

Professor Maxwell goes farther to point out the difficulties encountered by Sir William Thomson's Vortex Ring Theory of the constitution of matter; but he insists, as is seen from the above quotation, on the necessity of explaining mass and gravitation, as a requirement of a complete theory of matter. The assumption, therefore, that inertia is a property of matter requiring no explanation, is without warrant. While inertia is a property determined by experiment, it is doubtless a complex phenomenon requiring analysis; and as such it affords a lurking-place for much that is erroneous,—in a manner similar to the phenomenon of heat, which gave rise to the old, crude notion of phlogiston.

Finally, by a large class of mathematicians and physicists, force has been relegated to the shades of the antiquated, obsolete, to the realm of the unnecessary; or, as Herbert Spencer would say, perhaps, must be regarded as a pseud-idea.

Now, whether inertia and force are distinct from each other, or whether inertia be a phenomenon resulting from the action of force upon matter, the present state of knowledge will allow the expression of an opinion only. But a large number of physicists have claimed to account for all phenomena by the assumption of matter, ether, and motion, together with the absolute notions of space and time, as the only assumptions necessary. And the two notions which "cover a multitude of sins" in their philosophy are "inertia" and "potential energy."

Prof. P. G. Tait says in his article on "Mechanics" in the *Encyclopædia Britannica:*

> "Now it is impossible to conceive of a truly dormant form of energy whose magnitude should depend in any way on the unit of time; and we are therefore forced to the conclusion that potential energy, like kinetic energy, depends (in some yet unexplained, or rather unimagined, way) upon motion;" also, "The conclusion which appears inevitable is that whatever matter may be, the other reality in the physical universe which is never found unassociated with matter depends in all its widely varied forms upon motion of matter."

This is quoted approvingly by Prof. A. E. Dolbear in his *Matter, Ether, and Motion.* I do not find, however, that Professor Tait's co-laborer, Sir William Thomson (from an examination of their treatise on natural philosophy), has fallen into this error, so obvious when we consider the mathematical principles involved. Since the demonstration of the impossibility of such a position is so simple, it is the more surprising that so eminent a mathematician as Professor Tait should make this error. And since the same error has been made by various contributors to *The Monist*, some even assuming the conclusion of Professor Tait as an established principle of science, I will here present a brief discussion of the mathematical principles involved.

Assume the well-known formulæ (1) $M=mv$, and (2) $E=\frac{1}{2}mv^2$; in which m is the mass of matter (in the ordinary sense of the amount of matter), v its velocity of motion in any time unit, and M the momentum, or amount of motion; also E, the total energy of the body.

If we assume in (1) m a constant quantity, then M varies directly with v; in (2) the same assumption will show that E varies as v^2. Again, if we assume E constant (in accordance with the principle of conservation), then v^2, and consequently v, must be constant. That is, if there be a conservation of energy, there must be a conservation of motion. This, indeed, seems to be the contention of Prof. A. E. Dolbear as to motion in the universe.

Again, apply the same formulæ to a system of two non-elastic bodies before and after collision, assuming, for simplicity, all motion in one direction: (3) $M=mv+m'v'$, before collision, and (4) $M=\frac{1}{2}(m+m')(v+v')$, after collision; also (5) $E=\frac{1}{2}mv^2+\frac{1}{2}m'v'^2$, before collision, and (6) $E'=\frac{1}{2}(m+m')\frac{(v+v')^2}{4}$, after collision. Formulæ (3) and (4) are in accordance with the well-known principle that the momentum of the combined mass of two inelastic bodies is the same after collision as the sum of their momenta before collision; thus no motion is lost in collision. But from formulæ (5) and (6) we see that E and E' must in general be different in magnitude, and $E>E'$; that is, by collision some energy has disappeared. We also discover that the greatest disappearance of energy takes place when v is at a maximum and v' infinitely small, or when the body m is at rest. It is explained that the energy which has apparently disappeared has become potential by a change in the configuration of the system,—has done the work of changing the form of the bodies. But there is the same momentum after collision as before; there has been no loss of motion; it has been conserved in the system. Therefore, if potential energy be dependent on motion, whence the motion represented by the energy which has disappeared and become potential? This energy is represented by no motion whatever.

Again, if we assume two bodies moving toward each other in opposite directions, or any system of bodies colliding with directions of motion in any and all angles in space, there is a disappearance of both energy and motion. Such a system is resolvable into several, each system of two or more bodies having a common point of collision. We need, therefore, only to consider the case of a system of several bodies colliding in a common point to make the proposition general.

Let us assume, $mv \cos\phi + m'v' \cos\phi' + m''v'' \cos\phi''$ etc. $= M'$, in which ϕ, ϕ', ϕ'' represent, respectively, the angles which the lines of motion of the various bodies make with a directrix coincident with the resultant of M'. If $M' = 0$, then all motion and energy disappear. But if we assume the amount of motion in such a system to be a fixed quantity, then the energy of the system before collision will be at a minimum when $v = v' = v''$, etc.; and it will be at a maximum when there is a variation in the masses of the bodies and their velocities and when the smallest bodies have the highest velocity compatible with other conditions assumed. But when $v = v' = v''$, etc., collision causes a disappearance of a certain amount of motion and a certain amount of energy; and in any other case, that is, when v, v', and v'' are not equal, collision causes the disappearance of the same amount of motion as in the preceding case, but a greater amount of energy. Again, if M' in the equation assumed be not equal to zero, but be assumed constant, it is still obvious, if we assume the amount of motion in the system before collision to be constant, that the disappearance of motion on collision, when v, v', v', are caused to vary, is not attended by the disappearance of the same amount of energy.

It must appear, therefore, generally, from what precedes, that the amount of energy of a system may be increased or diminished by a mere change in configuration, the motion in the system remaining unchanged; and it is evident that this conclusion is inconsistent with the proposition that the energy which appears and disappears (the potential energy) is dependent upon motion. And we conclude, therefore, that, in a system of inelastic bodies, energy and motion cannot both be conserved; if energy be conserved, then motion cannot be. But if motion be not conserved and energy is, then some energy (potential energy) cannot be dependent on motion. It must also appear from the above given formulæ, that in any system where there is a change in the rate of motion, energy and motion cannot both be conserved; and if not both conserved, energy must pass from the kinetic state to potential, or *vice versa*, independently of motion. If motion of particles having high velocity be communicated to particles having a lower, energy becomes potential (is apparently lost) without the loss of motion. Or, if by any means the motion of particles having a low velocity could be communicated to particles having a higher, kinetic energy would appear without any change in the amount of motion, —would be represented by no motion.

Again, assume a system of perfectly elastic bodies, or infinitesimal particles, represented by the formulæ: $M = mv + m'v' + m''v''$, etc., and $E = \frac{1}{2}(mv^2 + m'v'^2 + m''v''^2 + \ldots .)$. Now in this problem the only case in which E and M are both conserved is that in which v, v', v'', etc., are constant. In this case there is a conservation of energy and of motion, and all energy is kinetic. But this condition could be fulfilled only in case the bodies were perfectly elastic and rebounds instantaneous. The latter condition is physically impossible; since in impact of two bodies some time must be required for compression, or distortion of form, and during which time the particles lose their motion and regain it. During impact there must be a change in the rate of motion; a loss of motion in such particles and a change of energy from kinetic to potential. In a gas the number of collisions occurring in a unit of time will be uniform, under the same conditions of mass, density, and temperature, and the proportion of kinetic to potential energy will be uniform. A gas, therefore, appears to be a system in which both energy and motion are conserved, since the molecules rebound after collision with the same velocity as that with which they approach. This, however, is apparent only; since at each collision there must be a variation in the velocity of the particles, and the moment a variation in velocity is introduced into the system motion and energy cannot appear and disappear proportionally and concomitantly. The same must hold true if the motion become vibratory, oscillatory, or rotary; since the velocity of any particle or part of any particle must then become variable. The only possible case in which a system of bodies can become conservative, both in energy and motion, is that in which every atom of matter and every material point of an atom, if we conceive an atom as made up of parts, shall have forever the same velocity it now has.

From the foregoing discussion it must appear that potential energy can be in no way dependent on motion; indeed, that this is true must be obvious from formulæ (1) and (2), and I have run the risk, in this discussion, of being criticised for attempting to prove what is evident, if we grant the truth of the formulæ. It is also evident, from the same formulæ, that kinetic energy is not wholly dependent on matter and motion; for, if the mass remain the same and be taken at unity, the energy will vary as the square of the amount of motion, whereas it should vary as the amount of motion, under the hypothesis that energy depend on the amount of matter and on motion as the only factors entering it. But if energy be not entirely dependent on motion and the amount of matter in the system, then it must, in part at least, be dependent on some factor not taken into consideration in the problem.

Such factor or factors must be force, or inertia, or both ; for there are no other factors adequate to the results observed.

Many inconsistencies would be avoided if writers on philosophic subjects would keep in view the elementary principles of mathematics, pure and applied, especially when discussing propositions which must have their foundations in mathematical principles. I cannot forbear criticising briefly, in this connexion, the article on "Vitalism," by Prof. C. Lloyd Morgan, which appeared in the January number of *The Monist*, and in which the writer was led into several absurdities. Two quotations only can be considered in this article. He says :

> "Prof. Burdon Sanderson wrote : 'The proof of the non-existence of a special "vital force" lies in the demonstration of the adequacy of the known sources of energy in the organism to account for the actual day by day expenditure of heat and work.' But an answer in due course came from the vitalists. It was pointed out that the application of a force to a moving body at right angles to its course alters the direction of motion without affecting its amount. The energy remains unchanged. Of such a directive character, it is sometimes urged, may be the application of vital force without presenting any phenomena contradictory of the generalisation that, in the operations of nature, energy is nowhere either destroyed or created."

Now the vitalists to whom Professor Morgan refers were ignorant of mechanics or he has misquoted them ; and in either case he seems not to have discovered, for some reason, the mathematical absurdity involved. Suppose a material point to move along the line *PA* with a velocity a ; then suppose at the point of application P a force be impressed on the point sufficient to give it a velocity b in the direction *PB*, at right angles to the motion in *PA*. Then the resultant motion will be in the direction *PR* with a velocity r, in magnitude $\sqrt{a^2+b^2}$; and if the mass be m, the energy will be $\frac{1}{2}m(a^2+b^2)$, instead of $\frac{1}{2}ma^2$, as before the force was impressed upon it.

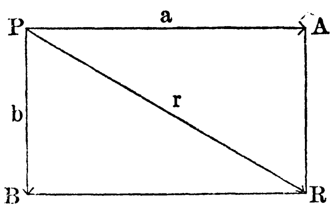

Fig. 1.

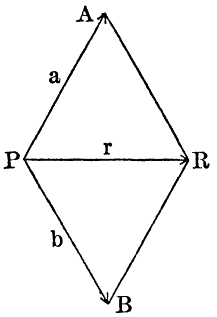

Fig. 2.

It will be observed that the resultant energy cannot be the same as before the force was impressed, except in case $b=0$, or that the force impressing the motion in *PB* be zero. In case $a=b=1$ and $m=1$, then the energy before the impression of the force will be $\frac{1}{2}$, while the energy of the resultant in *AR* will be 1. If however, we assume that $a = b$ and that the angle between their lines of direction be 120°, then $a = b = r$, and the motion and energy remain unchanged. But it is evident that r may have any value between the limits of zero and $a + b$, dependent on the variation of the angle between the lines of direction, PA and PB. And further, if we assume $a = r$, there is still an infinite number of values of b as determined by the angle APB, which will satisfy the condition that the motion of the material point and its energy shall be the same in magnitude after the impression of the force as before. It thus appears that the application of a force to a moving material point may leave its motion and energy unchanged, may increase the energy and motion, or may diminish them. This is a truth inconsistent with the conservation of an energy dependent on motion, though not inconsistent with the conservation of an energy reciprocally dependent on motion and force,—that is, partially dependent on motion and partially dependent on force, in such a manner that the sum of the two parts in any material system is a constant quantity.

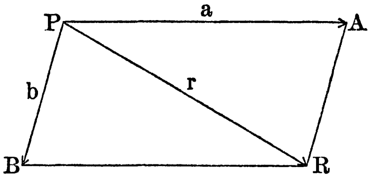

Fig. 3.

Professor Morgan continues :

> "So long as the metaphysical conceptions of force are carelessly commingled with the generalisations of dynamics as a science, this line of argument may appear to possess a cogency which is in truth fictitious. But what is the basal law of dynamics? That every movement of a part in any material system and every state of strain therein, has, as its antecedent, the assignable nature and distribution of the constituent parts of the system. This is a generalised statement of dynamic fact which quietly ignores (though it does not deny) the existence of force as the noumenal cause of motion. Granting therefore that a vital force is conceivable which alters the direction of motion without producing any change in the amount of energy, the question still remains: Is the movement so produced in accordance with, or is it contradictory to, the basal law of dynamics," etc.

To some this "basal law of dynamics" may be satisfactory ; but to the present writer it seems to assume by its terms the very propositions which it purports to explain. Let us look beneath its verbiage and see just what the "law" is, and how it comports with rationality. "Every movement and every state of strain has as its antecedent (cause) the assignable nature and distribution of the constituent parts of the system." Now what is the first antecedent or cause? "The assignable nature of the constituent parts of system." But what nature will we assign to these constituent parts? First, to explain motion we will assign to the particles of matter "inertia," the power to resist motion when the particle is at rest, and to continue in motion when it is in motion. We will assign gravitative attraction and molecular attractions and repulsions to account for gravitation, chemical affinity, adhesion, cohesion, and all motions (whether molecular or molar) which are the apparent results of such attractions and repulsions. We will assume material bonds, or hooks, to account for quantivalence of chemical atoms and the power to unite in multiple proportions. We will assume elasticity, the power to recover from distortion of form when the particle is subjected to strain. We will finally assign to a particle which is a constituent part of a system the power to act on other particles at a distance; which action most philosophers have considered not only impossible but inconceivable. In a word, we will beg the whole question by assigning just such a nature to the parts of our material system as will give us the results which we observe,—a most glaring case of the *petitio principii.* But this "assignable nature" must be fortified by the "distribution of the constituent parts of the system." That is, if one part is due east of another part, it will cause (be antecedent to) a certain motion or a certain strain (and what is a strain, if neither energy, force, nor motion?). But if one part relative to another be south, or east by south, or south by east, or east by south-southeast, or some one of an infinite number of other directions, then such distribution will cause, or be antecedent to, some other motion or some other strain. Or if one part be at a certain distance from some other part of the system, such distribution will cause, or be antecedent to, a different motion or strain than when the parts are located at other distances. Indeed, when Professor Morgan said (referring to his "basal law of dynamics"), that it "quietly ignores the existence of force," he should have told us what standing his "basal law" has in court that it should not be condemned as a generalised assumption,—one including under it many other assumptions with no *locus standi.*

There is too great a tendency on the part of many writers to assume certain generalised statements of observed phenomena as containing their own sufficient explanation. The law of gravity is a generalisation which nearly all men accept as true; yet it carries no sufficient explanation of the more fundamental propositions in it involved. The doctrine of conservation of energy is a statement of a certain observed uniformity of nature; but it has in itself no adequate explanation of the various more elementary truths involved in it. We are not even able to say what these more elementary truths are.

ITHACA, MICH. CHAS. H. CHASE.